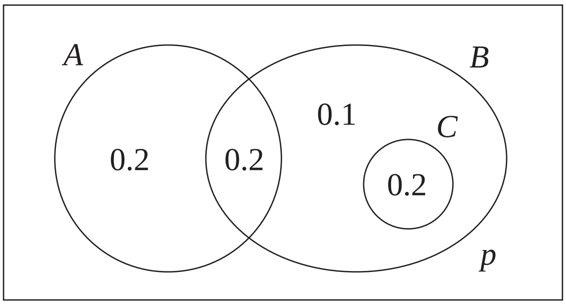
1.



The Venn diagram, where p is a probability, shows the 3 events A, B and C with their associated probabilities.

(a) Find the value of p.

(1)

(b) Write down a pair of mutually exclusive events from A, B and C.

(1)

a) Probabilities all add to 1

$0.2 + 0.2 + 0.1 + 0.2 + p = 1$

$p = 1 - (0.2 + 0.2 + 0.1 + 0.2)$

$p = 0.3$ (1)

b) mutually exclusive = cannot happen at the same time

⇒ no overlap in Venn diagram

A and C are mutually exclusive. (1)

2. Two bags, **A** and **B**, each contain balls which are either red or yellow or green.

Bag **A** contains 4 red, 3 yellow and n green balls.
Bag **B** contains 5 red, 3 yellow and 1 green ball.

A ball is selected at random from bag **A** and placed into bag **B**.
A ball is then selected at random from bag **B** and placed into bag **A**.

The probability that bag **A** now contains an equal number of red, yellow and green balls is p.

Given that $p > 0$, find the possible values of n and p.

(5)

must end up with 3 or 4 of each colour in A. (1)

$\therefore n \leq 5$ as if n was 6 or more, we couldn't get the number of green balls in A down to 4.

$n \geq 2$ as if n was 1 or 0 we couldnt get the number of green balls in A up to 3.

if $n=2$: requires red from $A \to B$ and green from $B \to A$ (1)

$= \frac{4 \text{ red}}{9 \text{ total}} \times \frac{1 \text{ green}}{10 \text{ total}} = \frac{2}{45} \quad \therefore p = \frac{2}{45}$ (1)

9 balls were originally in B, then 1 was added

if $n=3$, no way to make equal number of all colours
if $n=4$, ditto

if $n=5$: requires green from $A \to B$ and yellow from $B \to A$ (1)

$= \frac{5 \text{ green}}{12 \text{ total}} \times \frac{3 \text{ yellow}}{10 \text{ total}} = \frac{1}{8} \quad \therefore p = \frac{1}{8}$ (1)

3. A manufacturer of sweets knows that 8% of the bags of sugar delivered from supplier *A* will be damp.
A random sample of 35 bags of sugar is taken from supplier *A*.

(a) Using a suitable model, find the probability that the number of bags of sugar that are damp is

(i) exactly 2

(ii) more than 3

(3)

Supplier *B* claims that when it supplies bags of sugar, the proportion of bags that are damp is less than 8%

The manufacturer takes a random sample of 70 bags of sugar from supplier *B* and finds that only 2 of the bags are damp.

(b) Carry out a suitable test to assess supplier *B*'s claim.
You should state your hypotheses clearly and use a 10% level of significance.

(4)

a) Let D = number of bags that are damp from A

$D \sim B(35, 0.08)$ 1

(i) $P(D=2) = 0.243$ (3 s.f.) 1

$P(D>3) = 1 - P(D \leqslant 3)$

$= 1 - 0.6939... = 0.306$ (3 s.f.) 1

b) Let X : number of bags that are damp from B

$X \sim B(70, 0.08)$ 1

$H_0 : p = 0.08$, $H_1 : p < 0.08$ 1

$\alpha = 0.1$

$P(X \leqslant 2) = 0.0740$ (3 s.f.) 1

0.0740 is < 0.1 , so reject H_0 since sufficient evidence to support B's claim

4. Manon has two biased spinners, one red and one green.

The random variable R represents the score when the red spinner is spun.
The random variable G represents the score when the green spinner is spun.

The probability distributions for R and G are given below.

r	2	3
$P(R = r)$	$\frac{1}{4}$	$\frac{3}{4}$

g	1	4
$P(G = g)$	$\frac{2}{3}$	$\frac{1}{3}$

Manon spins each spinner once and adds the two scores.

(a) Find the probability that

(i) the sum of the two scores is 7

(ii) the sum of the two scores is less than 4

(3)

The random variable $X = mR + nG$ where m and n are integers.

$$P(X = 20) = \frac{1}{6} \qquad \text{and} \qquad P(X = 50) = \frac{1}{4}$$

(b) Find the value of m and the value of n

(5)

a) (i) The sum of the two scores is 7 when:

$R = 3$, $G = 4$

$\therefore P(\text{two scores is } 7) = \frac{3}{4} \times \frac{1}{3}$ 1

$= \frac{1}{4}$ 1

(ii) The sum of the two scores is less than 4 when:

$R = 2$, $G = 1$

$\therefore P(\text{two scores is less than } 4) = \frac{1}{4} \times \frac{2}{3}$

$= \frac{1}{6}$ 1

from (a)

b) $P(x=50) = \frac{1}{4}$ $\therefore$ means $R=3$ and $G=4$ 1

from (a)

$P(x=20) = \frac{1}{6}$ $\therefore$ means $R=2$ and $G=1$

So for $P(x=50)$: $50 = 3m + 4n$ — ① 1

for $P(x=20)$: $20 = 2m + n$ — ② 1

substitute ② into ①

$50 = 3m + 4(20 - 2m)$ 1

$= 3m + 80 - 8m$

$5m = 30$

$m = 6$ 1

substitute $m = 6$ into ②

$20 = 2(6) + n$

$n = 8$

5. In an after-school club, students can choose to take part in Art, Music, both or neither.

There are 45 students that attend the after-school club. Of these

- 25 students take part in Art
- 12 students take part in both Art and Music
- the number of students that take part in Music is x

(a) Find the range of possible values of x

(2)

One of the 45 students is selected at random.

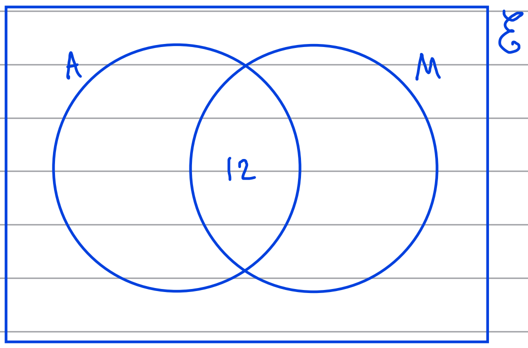
Event A is the event that the student selected takes part in Art.

Event M is the event that the student selected takes part in Music.

(b) Determine whether or not it is possible for the events A and M to be independent.

(4)

a)



Art only: $25 - 12 = 13$ ①

Music only: $x - 12$

Neither: $45 - (13 + 12 + x - 12)$
$= 45 - (13 + x)$
$= 32 - x$

$x - 12 \geq 0 \Rightarrow x \geq 12$

$32 - x \geq 0 \Rightarrow x \leq 32$

$\therefore \quad 12 \leq x \leq 32$ ①

b) For independence, $P(A)P(M) = P(A \cap M)$ ①

$P(A) = \frac{25}{45}$ $\quad \therefore P(M) = \frac{12/45}{25/45} = \frac{12}{25}$ ①

$P(A \cap M) = \frac{12}{45}$ $\quad$ students taking music $= \frac{12}{25} \times 45 = 21.6$ ①

21.6 is not a whole number, so A and M cannot be independent. ①

6. Julia selects 3 letters at random, one at a time without replacement, from the word

VARIANCE 8 letters

The discrete random variable X represents the number of times she selects a letter A.

(a) Find the complete probability distribution of X.

(5)

Yuki selects 10 letters at random, one at a time **with** replacement, from the word

DEVIATION

(b) Find the probability that he selects the letter E at least 4 times.

(3)

a) A appears twice so $X = 0, 1$ or 2 ①

$$P(X=0) = \frac{6}{8} \times \frac{5}{7} \times \frac{4}{6} = \frac{5}{14} \quad ①$$

$$P(X=1) = 3\left(\frac{2}{8} \times \frac{6}{7} \times \frac{5}{6}\right) = \frac{15}{28}$$

$$P(X=2) = 3\left(\frac{2}{8} \times \frac{1}{7} \times \frac{6}{6}\right) = \frac{3}{28} \quad ①$$

x	0	1	2
$P(X=x)$	$5/14$	$15/28$	$3/28$

②

if A = choosing A
A' = NOT choosing A

choosing A once: A, A', A' OR A', A, A' OR A', A', A

these events all have the same probability.

choosing A twice: A, A, A' OR A, A', A OR A', A, A

these events all have the same probability.

b) Deviation = 9 letters and 1 E. $\therefore P(\text{choose } E) = \frac{1}{9}$

let X = # of times E is selected

$X \sim B(10, 1/9)$ 1

$P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9816 = 0.0184$ (4dp) 1 1

7. (a) State one disadvantage of using quota sampling compared with simple random sampling.

(1)

In a university 8% of students are members of the university dance club.

A random sample of 36 students is taken from the university.

The random variable X represents the number of these students who are members of the dance club.

(b) Using a suitable model for X, find

(i) $P(X = 4)$

(ii) $P(X \geqslant 7)$

(3)

Only 40% of the university dance club members can dance the tango.

(c) Find the probability that a student is a member of the university dance club and can dance the tango.

(1)

A random sample of 50 students is taken from the university.

(d) Find the probability that fewer than 3 of these students are members of the university dance club and can dance the tango.

(2)

a) Disadvantage of quota sampling compared with simple random sampling :

$\Rightarrow$ Not random (1)

b) $X \sim B(36, 0.08)$ (1)

(i) $P(X=4) = 0.167387\ldots$

$= 0.167$ (3 s.f.) (1)

(ii) $P(X \geqslant 7) = 1 - P(X \leqslant 6)$

$= 1 - 0.977\ldots$

$= 0.022233\ldots$

$= 0.0222$ (3 s.f.) (1)

(c) $P(\text{dance club} \cap \text{dance tango}) = 0.08 \times 0.4$
$= 0.032$ (1)

d) let T = dance club and dance tango

$T \sim B(50, 0.032)$ (1)

$P(T < 3) = P(T \leq 2)$

$= 0.785081\ldots$
$= 0.785$ (3 s.f.) (1)

8. The discrete random variable X has the following probability distribution

x	a	b	c
$P(X = x)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

where

- a, b and c are distinct integers $(a < b < c)$
- all the probabilities are greater than zero

(a) Find

(i) the value of a

(ii) the value of b

(iii) the value of c

Show your working clearly.

(5)

The independent random variables X_1 and X_2 each have the same distribution as X

(b) Find $P(X_1 = X_2)$

(2)

a) $\sum$ probabilities $= 1$: $\log_{36} a + \log_{36} b + \log_{36} c = 1$ (1)

$\log_{36} abc = 1$

$abc = 36^1$

$\therefore abc = 36$ (1)

All probabilities are greater than zero :

Hence, $a, b, c > 1$, since $\log_{36} 1 = 0$ (1)

$\Rightarrow$ if we take factor of 36, and a, b, c are distinct integers.

Smallest factor $= 2$. So, $a = 2$ (1)

Next smallest factor $= 3$. So, $b = 3$

$c = 36 \div (2 \times 3) = 6$ (1)

b)

x	a	b	c
$P(X_1)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$
$P(X_2)$	$\log_{36} a$	$\log_{36} b$	$\log_{36} c$

$P(X_1 = X_2) = (\log_{36} a)^2 + (\log_{36} b)^2 + (\log_{36} c)^2$ (1)

$= (\log_{36} 2)^2 + (\log_{36} 3)^2 + (\log_{36} 6)^2$

$= 0.0374137... + 0.0939873\dot{7} + 0.25$

$= 0.38140...$

$= 0.381$ (3 s.f.) (1)

9. A company has 1825 employees.
The employees are classified as professional, skilled or elementary.

The following table shows

- the number of employees in each classification
- the two areas, A or B, where the employees live

	A	B
Professional	740	380
Skilled	275	90
Elementary	260	80

An employee is chosen at random.

Find the probability that this employee

(a) is skilled,
(1)

(b) lives in area B and is not a professional.
(1)

Some classifications of employees are more likely to work from home.

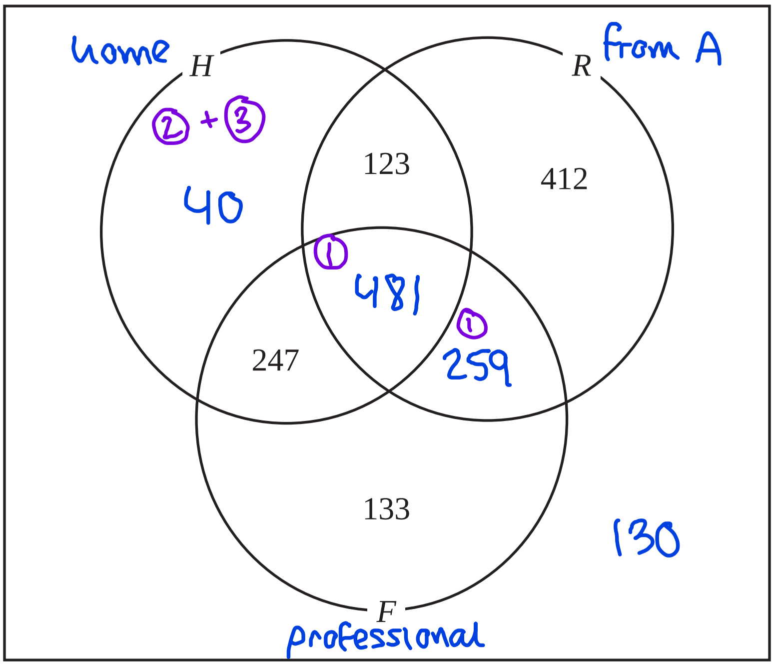
- 1 65% of professional employees in both area A and area B work from home
- 2 40% of skilled employees in both area A and area B work from home
- 3 5% of elementary employees in both area A and area B work from home
- Event F is that the employee is a professional
- Event H is that the employee works from home
- Event R is that the employee is from area A

(c) Using this information, complete the Venn diagram on the opposite page.
(4)

(d) Find $\mathrm{P}(R' \cap F)$
(1)

(e) Find $\mathrm{P}([H \cup R]')$
(1)

(f) Find $\mathrm{P}(F \mid H)$
(2)



① ① ①

Turn over for a spare diagram if you need to redraw your Venn diagram.

a) $P(\text{skilled}) = \dfrac{275+90}{1825} = \dfrac{1}{5}$ ①

b) $P(\text{B and not professional}) = \dfrac{90+80}{1825} = \dfrac{34}{365}$ ①

c) ① 740 professional from A, 65% work from home
$740 \times 0.65 = 481$ ①

② 90 skilled from B, 40% work from home
$90 \times 0.4 = 36$

③ 80 elementary from B, 5% work from home
$80 \times 0.05 = 4$

$36 + 4 = 40$ non professionals from B who work from home

① 740 professional from A, 35% do not work from home: $740 \times 0.35 = 259$

$1825 - (40 + 123 + 412 + 481 + 247 + 259 + 133)$

$= 130$

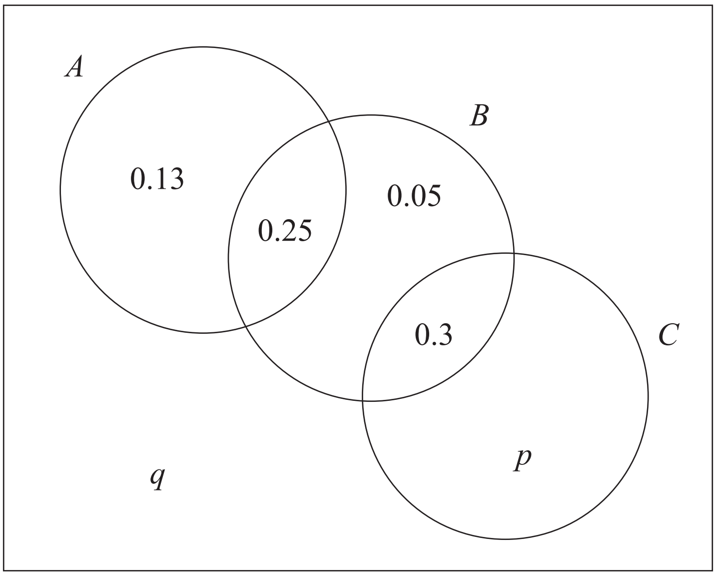
d) $P(R' \cap F) = \dfrac{247 + 133}{1825} = 0.208 \ (3sf)$ (1)

e) $P([H \cup R]') = \dfrac{133 + 130}{1825} = 0.144 \ (3sf)$ (1)

f) $P(F \mid H) = \dfrac{P(F \cap H)}{P(H)} = \dfrac{247 + 481}{40 + 123 + 247 + 481}$ (1)

$= 0.817 \ (3sf)$ (1)

10. The Venn diagram, where p and q are probabilities, shows the three events A, B and C and their associated probabilities.



(a) Find P(A)

(1)

The events B and C are independent.

(b) Find the value of p and the value of q

(3)

(c) Find $P(A|B')$ → $\dfrac{P(A \cap B')}{P(B')}$

(2)

a) $P(A) = 0.13 + 0.25$

$= 0.38$ (1)

b) $P(B \cap C) = P(B) \times P(C)$ — independent event

$0.3 = (0.3 + 0.25 + 0.05) \times (0.3 \times p)$ (1)

$0.3 = 0.6 \times (0.3 + p)$

$0.3 = 0.18 + 0.6p$

$0.12 = 0.6p$

$\therefore p = 0.2$ (1)

Σ probabilities $= 1$:

$0.13 + 0.25 + 0.05 + 0.3 + p + q = 1$

$0.13 + 0.25 + 0.05 + 0.3 + 0.2 + q = 1$

$0.93 + q = 1$

$\therefore q = 0.07$ (1)

(c) $P(A|B') = \dfrac{P(A \cap B')}{P(B')}$

$= \dfrac{0.13}{0.13 + 0.2 + 0.07}$ (1)

$= \dfrac{0.13}{0.4}$

$= 0.325$ (1)